NAG C Library Function Document

nag_ztrtrs (f07tsc)

1 Purpose

nag_ztrtrs (f07tsc) solves a complex triangular system of linear equations with multiple right-hand sides, $AX = B$, $A^T X = B$ or $A^H X = B$.

2 Specification

```
void nag_ztrtrs (Nag_OrderType order, Nag_UploType uplo, Nag_TransType trans,
     Nag_DiagType diag, Integer n, Integer nrhs, const Complex a[], Integer pda,
     Complex b[], Integer pdb, NagError *fail)
```

3 Description

nag_ztrtrs (f07tsc) solves a complex triangular system of linear equations $AX = B$, $A^T X = B$ or $A^H X = B$.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

Higham N J (1989) The accuracy of solutions to triangular systems *SIAM J. Numer. Anal.* **26** 1252–1265

5 Parameters

1: **order** – Nag_OrderType *Input*

On entry: the **order** parameter specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = **Nag_RowMajor**. See Section 2.2.1.4 of the Essential Introduction for a more detailed explanation of the use of this parameter.

Constraint: **order** = **Nag_RowMajor** or **Nag_ColMajor**.

2: **uplo** – Nag_UploType *Input*

On entry: indicates whether A is upper or lower triangular as follows:

if **uplo** = **Nag_Upper**, A is upper triangular;

if **uplo** = **Nag_Lower**, A is lower triangular.

Constraint: **uplo** = **Nag_Upper** or **Nag_Lower**.

3: **trans** – Nag_TransType *Input*

On entry: indicates the form of the equations as follows:

if **trans** = **Nag_NoTrans**, then the equations are of the form $AX = B$;

if **trans** = **Nag_Trans**, then the equations are of the form $A^T X = B$;

if **trans** = **Nag_ConjTrans**, then the equations are of the form $A^H X = B$.

Constraint: **trans** = **Nag_NoTrans**, **Nag_Trans** or **Nag_ConjTrans**.

4: **diag** – Nag_DiagType *Input*

On entry: indicates whether A is a non-unit or unit triangular matrix as follows:

if **diag** = **Nag_NonUnitDiag**, then A is a non-unit triangular matrix;

if **diag** = **Nag_UnitDiag**, then A is a unit triangular matrix; the diagonal elements are not referenced and are assumed to be 1.

Constraint: **diag** = **Nag_NonUnitDiag** or **Nag_UnitDiag**.

5: **n** – Integer *Input*

On entry: n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

6: **nrhs** – Integer *Input*

On entry: r, the number of right-hand sides.

Constraint: $\mathbf{nrhs} \geq 0$.

7: **a**[dim] – const Complex *Input*

Note: the dimension, dim, of the array **a** must be at least $\max(1, \mathbf{pda} \times \mathbf{n})$.

On entry: the n by n triangular matrix A. If **uplo** = **Nag_Upper**, A is upper triangular and the elements of the array below the diagonal are not referenced; if **uplo** = **Nag_Lower**, A is lower triangular and the elements of the array above the diagonal are not referenced. If **diag** = **Nag_UnitDiag**, the diagonal elements of A are not referenced, but are assumed to be 1.

8: **pda** – Integer *Input*

On entry: the stride separating row or column elements (depending on the value of **order**) of the matrix A in the array **a**.

Constraint: $\mathbf{pda} \geq \max(1, \mathbf{n})$.

9: **b**[dim] – Complex *Input/Output*

Note: the dimension, dim, of the array **b** must be at least $\max(1, \mathbf{pdb} \times \mathbf{nrhs})$ when **order** = **Nag_ColMajor** and at least $\max(1, \mathbf{pdb} \times \mathbf{n})$ when **order** = **Nag_RowMajor**.

If **order** = **Nag_ColMajor**, the (i, j)th element of the matrix B is stored in $\mathbf{b}[(j-1) \times \mathbf{pdb} + i - 1]$ and if **order** = **Nag_RowMajor**, the (i, j)th element of the matrix B is stored in $\mathbf{b}[(i-1) \times \mathbf{pdb} + j - 1]$.

On entry: the n by r right-hand side matrix B.

On exit: the n by r solution matrix X.

10: **pdb** – Integer *Input*

On entry: the stride separating matrix row or column elements (depending on the value of **order**) in the array **b**.

Constraints:

if **order** = **Nag_ColMajor**, $\mathbf{pdb} \geq \max(1, \mathbf{n})$;
if **order** = **Nag_RowMajor**, $\mathbf{pdb} \geq \max(1, \mathbf{nrhs})$.

11: **fail** – NagError * *Output*

The NAG error parameter (see the Essential Introduction).

6 Error Indicators and Warnings

NE_INT

On entry, **n** = ⟨*value*⟩.
Constraint: **n** ≥ 0.

On entry, **nrhs** = ⟨*value*⟩.
Constraint: **nrhs** ≥ 0.

On entry, **pda** = ⟨*value*⟩.
Constraint: **pda** > 0.

On entry, **pdb** = ⟨*value*⟩.
Constraint: **pdb** > 0.

NE_INT_2

On entry, **pda** = ⟨*value*⟩, **n** = ⟨*value*⟩.
Constraint: **pda** $\geq \max(1, \mathbf{n})$.

On entry, **pdb** = ⟨*value*⟩, **n** = ⟨*value*⟩.
Constraint: **pdb** $\geq \max(1, \mathbf{n})$.

On entry, **pdb** = ⟨*value*⟩, **nrhs** = ⟨*value*⟩.
Constraint: **pdb** $\geq \max(1, \mathbf{nrhs})$.

NE_SINGULAR

$a(\langle value \rangle, \langle value \rangle)$ is zero, and the matrix A is singular.

NE_ALLOC_FAIL

Memory allocation failed.

NE_BAD_PARAM

On entry, parameter ⟨*value*⟩ had an illegal value.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

7 Accuracy

The solutions of triangular systems of equations are usually computed to high accuracy. See Higham (1989).

For each right-hand side vector b, the computed solution x is the exact solution of a perturbed system of equations $(A + E)x = b$, where

$$|E| \leq c(n)\epsilon|A|,$$

$c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***.

If $\hat{x}$ is the true solution, then the computed solution x satisfies a forward error bound of the form

$$\frac{\|x - \hat{x}\|_\infty}{\|x\|_\infty} \leq c(n)\,\mathrm{cond}(A, x)\epsilon,$$

provided $c(n)\,\mathrm{cond}(A, x)\epsilon < 1$, where $\mathrm{cond}(A, x) = \||A^{-1}|\,|A|\,|x|\|_\infty / \|x\|_\infty$.

Note that $\mathrm{cond}(A, x) \leq \mathrm{cond}(A) = \||A^{-1}|\,|A|\|_\infty \leq \kappa_\infty(A)$; $\mathrm{cond}(A, x)$ can be much smaller than $\mathrm{cond}(A)$ and it is also possible for $\mathrm{cond}(A^H)$, which is the same as $\mathrm{cond}(A^T)$, to be much larger (or smaller) than $\mathrm{cond}(A)$.

Forward and backward error bounds can be computed by calling nag_ztrrfs (f07tvc), and an estimate for $\kappa_\infty(A)$ can be obtained by calling nag_ztrcon (f07tuc) with **norm** = **Nag_InfNorm**.

8 Further Comments

The total number of real floating-point operations is approximately $4n^2r$.

The real analogue of this function is nag_dtrtrs (f07tec).

9 Example

To solve the system of equations $AX = B$, where

$$A = \begin{pmatrix} 4.78 + 4.56i & 0.00 + 0.00i & 0.00 + 0.00i & 0.00 + 0.00i \\ 2.00 - 0.30i & -4.11 + 1.25i & 0.00 + 0.00i & 0.00 + 0.00i \\ 2.89 - 1.34i & 2.36 - 4.25i & 4.15 + 0.80i & 0.00 + 0.00i \\ -1.89 + 1.15i & 0.04 - 3.69i & -0.02 + 0.46i & 0.33 - 0.26i \end{pmatrix}$$

and

$$B = \begin{pmatrix} -14.78 - 32.36i & -18.02 + 28.46i \\ 2.98 - 2.14i & 14.22 + 15.42i \\ -20.96 + 17.06i & 5.62 + 35.89i \\ 9.54 + 9.91i & -16.46 - 1.73i \end{pmatrix}.$$

9.1 Program Text

```
/* nag_ztrtrs (f07tsc) Example Program.
 *
 * Copyright 2001 Numerical Algorithms Group.
 *
 * Mark 7, 2001.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf07.h>
#include <nagx04.h>

int main(void)
{
  /* Scalars */
  Integer i, j, n, nrhs, pda, pdb;
  Integer exit_status=0;
  Nag_UploType uplo_enum;

  NagError fail;
  Nag_OrderType order;
  /* Arrays */
  char    uplo[2];
  Complex *a=0, *b=0;

#ifdef NAG_COLUMN_MAJOR
#define A(I,J) a[(J-1)*pda + I - 1]
#define B(I,J) b[(J-1)*pdb + I - 1]
  order = Nag_ColMajor;
#else
#define A(I,J) a[(I-1)*pda + J - 1]
#define B(I,J) b[(I-1)*pdb + J - 1]
  order = Nag_RowMajor;
#endif

  INIT_FAIL(fail);
  Vprintf("f07tsc Example Program Results\n\n");

  /* Skip heading in data file */
```

```
  Vscanf("%*[^\n] ");
  Vscanf("%ld%ld%*[^\n] ", &n, &nrhs);
#ifdef NAG_COLUMN_MAJOR
  pda = n;
  pdb = n;
#else
  pda = n;
  pdb = nrhs;
#endif

  /* Allocate memory */
  if ( !(a = NAG_ALLOC(n * n, Complex)) ||
       !(b = NAG_ALLOC(n * nrhs, Complex)) )
    {
      Vprintf("Allocation failure\n");
      exit_status = -1;
      goto END;
    }

  /* Read A and B from data file */
  Vscanf(" ' %1s '%*[^\n] ", uplo);
  if (*(unsigned char *)uplo == 'L')
    uplo_enum = Nag_Lower;
  else if (*(unsigned char *)uplo == 'U')
    uplo_enum = Nag_Upper;
  else
    {
      Vprintf("Unrecognised character for Nag_UploType type\n");
      exit_status = -1;
      goto END;
    }
  if (uplo_enum == Nag_Upper)
    {
      for (i = 1; i <= n; ++i)
        {
          for (j = i; j <= n; ++j)
            Vscanf(" ( %lf , %lf )", &A(i,j).re, &A(i,j).im);
        }
      Vscanf("%*[^\n] ");
    }
  else
    {
      for (i = 1; i <= n; ++i)
        {
          for (j = 1; j <= i; ++j)
            Vscanf(" ( %lf , %lf )", &A(i,j).re, &A(i,j).im);
        }
      Vscanf("%*[^\n] ");
    }
  for (i = 1; i <= n; ++i)
    {
      for (j = 1; j <= nrhs; ++j)
        Vscanf(" ( %lf , %lf )", &B(i,j).re, &B(i,j).im);
    }
  Vscanf("%*[^\n] ");

  /* Compute solution */
  f07tsc(order, uplo_enum, Nag_NoTrans, Nag_NonUnitDiag, n,
         nrhs, a, pda, b, pdb, &fail);
  if (fail.code != NE_NOERROR)
    {
      Vprintf("Error from f07tsc.\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }

  /* Print solution */
  x04dbc(order, Nag_GeneralMatrix, Nag_NonUnitDiag, n, nrhs,
         b, pdb, Nag_BracketForm,"%7.4f", "Solution(s)",
         Nag_IntegerLabels, 0, Nag_IntegerLabels, 0, 80, 0,
         0, &fail);
```

```
  if (fail.code != NE_NOERROR)
    {
      Vprintf("Error from x04dbc.\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }

 END:
  if (a) NAG_FREE(a);
  if (b) NAG_FREE(b);

  return exit_status;
}
```

9.2 Program Data

```
f07tsc Example Program Data
  4  2                                                        :Values of N and NRHS
  'L'                                                         :Value of UPLO
 ( 4.78, 4.56)
 ( 2.00,-0.30) (-4.11, 1.25)
 ( 2.89,-1.34) ( 2.36,-4.25) ( 4.15, 0.80)
 (-1.89, 1.15) ( 0.04,-3.69) (-0.02, 0.46) ( 0.33,-0.26)  :End of matrix A
 (-14.78,-32.36) (-18.02, 28.46)
 (  2.98, -2.14) ( 14.22, 15.42)
 (-20.96, 17.06) (  5.62, 35.89)
 (  9.54,  9.91) (-16.46, -1.73)                              :End of matrix B
```

9.3 Program Results

```
f07tsc Example Program Results

 Solution(s)
                    1                   2
 1  (-5.0000,-2.0000)  ( 1.0000, 5.0000)
 2  (-3.0000,-1.0000)  (-2.0000,-2.0000)
 3  ( 2.0000, 1.0000)  ( 3.0000, 4.0000)
 4  ( 4.0000, 3.0000)  ( 4.0000,-3.0000)
```